

Learning to Reason with HOL4 Tactics

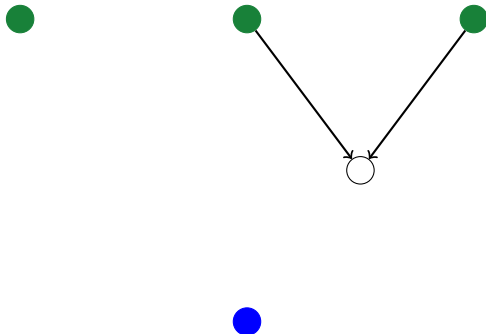
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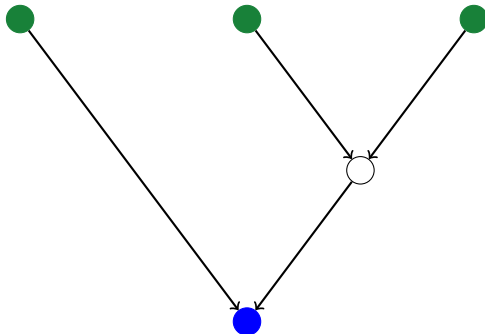
Reasoning with inference rules



Reasoning with inference rules



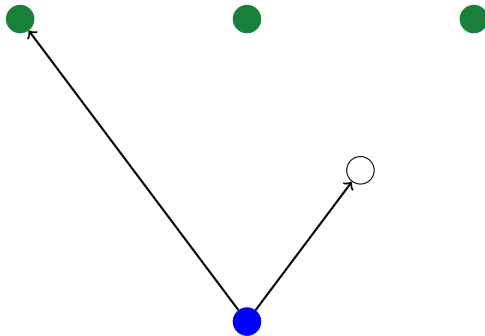
Reasoning with inference rules



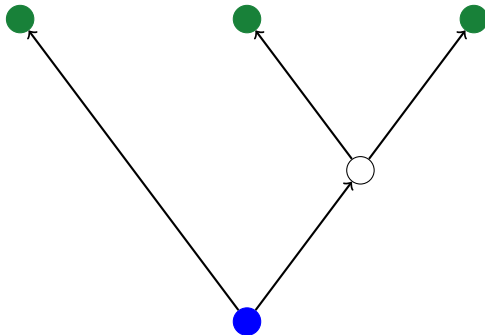
Reasoning with tactics



Reasoning with tactics



Reasoning with tactics



Tactics

REWRITE_TAC

INDUCT_TAC

METIS_TAC

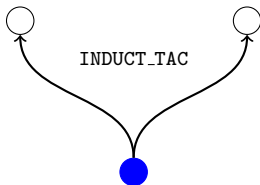
Composing tactics

THENL tactical composes the effect of tactics.



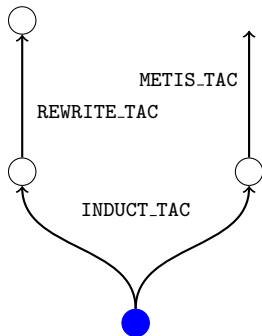
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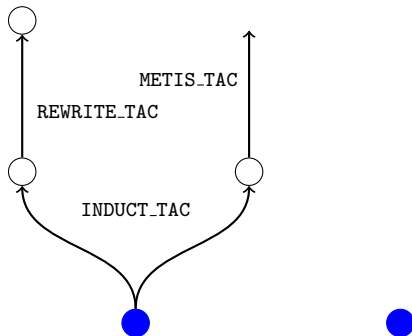
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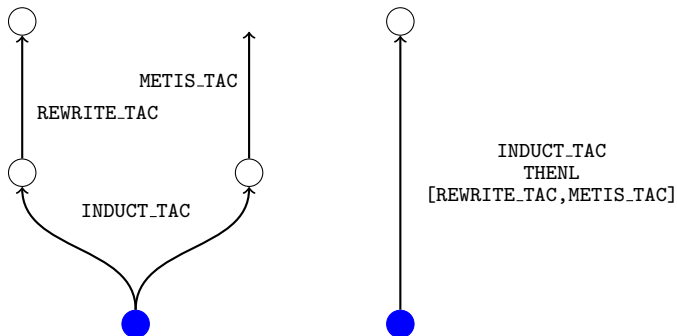
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Composing tactics

THENL tactical composes the effect of tactics.



Tactic selection

Was the tactic successful **before** on **similar** goals?

Before: recording tactics

Globalization:

```
val x = 2 + 2; val tac = NTAC x REWRITE_TAC;
```

tac becomes

```
Tactical.NTAC (2 + 2) Ho_rewrite.REWRITE_TAC;
```

Recording function:

```
(R INDUCT_TAC) THENL [R REWRITE_TAC, R METIS_TAC]
```

Feature vectors:

"INDUCT_TAC" $x + x \geq x$

"REWRITE_TAC" $0 + 0 \geq 0$

"METIS_TAC" $x + x \geq x \implies S x + S x \geq S x$

Similar: predicting tactics

Feature vectors:

"INDUCT_TAC" $x + x \geq x$

"REWRITE_TAC" $0 + 0 \geq 0$

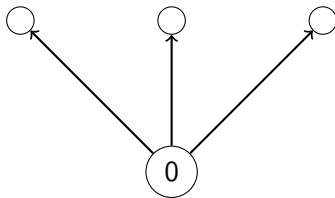
"METIS_TAC" $x + x \geq x \implies S\ x + S\ x \geq S\ x$

Features: subterms

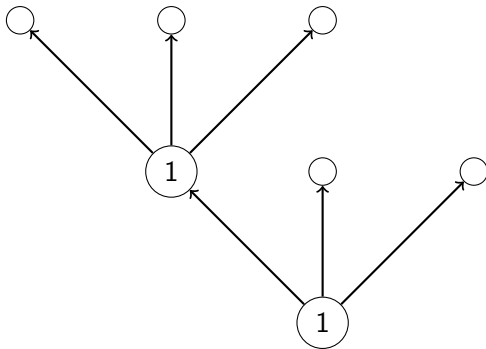
Ordering tactics for $x * x \geq x$? (**policy**)

[INDUCT_TAC 0.9, METIS_TAC 0.89, REWRITE_TAC 0.5]

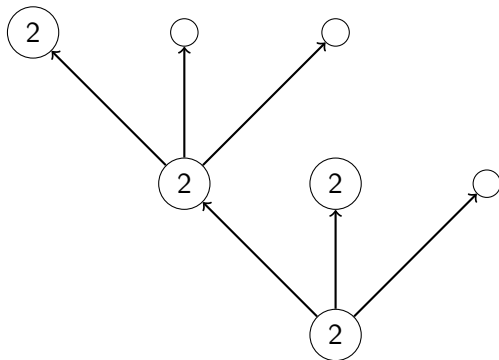
A*-search: Cost (**evaluation**)



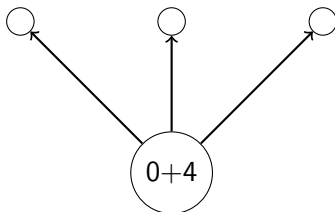
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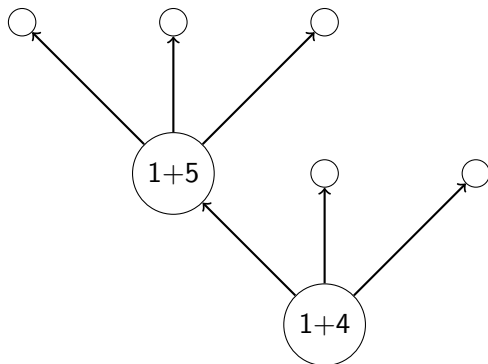
A*-search: Cost (**evaluation**)



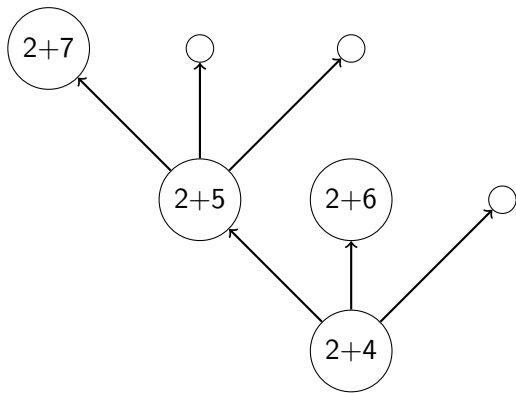
A*-search: Cost + Heuristic (**evaluation**)



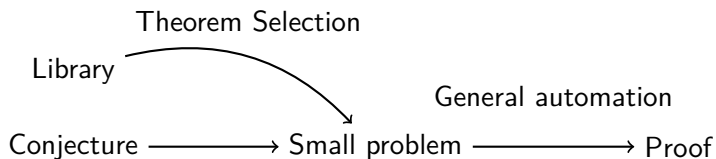
A*-search: Cost + Heuristic (**evaluation**)



A*-search: Cost + Heuristic (**evaluation**)



Hammers



Re-proving the HOL4 standard library

Prover (5 seconds)	7902 theorems
TacticToe	29.73
HolyHammer [Eprover blistr]	32.35
TacticToe [Metis]	39.42
TacticToe [Metis] (New version)	45.73

Techniques

Heuristic (**evaluation**): average proof length of similar provable goals.

Orthogonalization (**policy**): avoid applying a tactic that have the same effect on a goal than a previous one.

Minimization and prettification (**user interaction**): produces faster proofs and readable proofs.

Contingent weaknesses

Incompleteness: compensated by calls to Metis.

HolyHammer is better in dense theories.

Tactics are slow: 0.02 seconds per calls.

Example

$$\forall ls\ n. (DROP\ n\ ls = []) \Leftrightarrow n \geq LENGTH\ ls$$

Induct THEN SRW_TAC [] [] THEN DECIDE_TAC

Induct THENL [SRW_TAC [] [], SRW_TAC [ARITH_ss] []]

Demo

Conclusion

TacticToe combines previous human proofs to solve new goals.

- ▶ Induction principles
- ▶ Simplification sets
- ▶ User-defined domain specific automation
- ▶ General automation

The proofs produced are “efficient” HOL4 proof scripts.

How should we make the magic happen?

Recording, Data preprocessing

Policy, Evaluation

Reinforcement learning, Tactic synthesis