Recovering Formal Representations from Informalized Mizar Sentences

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Outline

Introduction

- Long term goal: tools to read informal mathematics and create formalized versions.
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- Stepping stone: handle cases where we already have the formal versions
  - Flyspeck: Informal document by Hales; Formalization in HOL-light.
  - MML (Mizar Mathematical Library): Mizar looks similar to informal mathematics, but is essentially formal since Mizar can check it.

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  - Here $Y \subseteq X$ and $Y \subseteq X'$.
  - Two readings:
    - Take $\overline{Y}$ to mean $X \setminus Y$. (false)
    - Take $\overline{Y}$ to mean $X' \setminus Y$. (true)

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- The text looks potentially ambiguous, but is resolved by the environment given in the header of the Mizar article.
- Annotated XML versions obtained when Mizar checks the articles, creating versions independent of the article header.
- From the XML versions various representations can be derived:
  - Parse trees
  - "Pattern representations" (partial first-order terms and propositions with missing information)
  - (MPTP) "Constructor representations" (full unambiguous first-order terms and propositions)
  - MPTP has been used for first order LTB competitions; higher order version coming soon

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  - Constructor representations of the MML definitions and theorems.
- Use these terms/propositions to interact with theorem provers
  - Call theorem provers on the constructor representations of the MML theorems.

Outline

Introduction

Parsing

Mizar Typing System

Examples

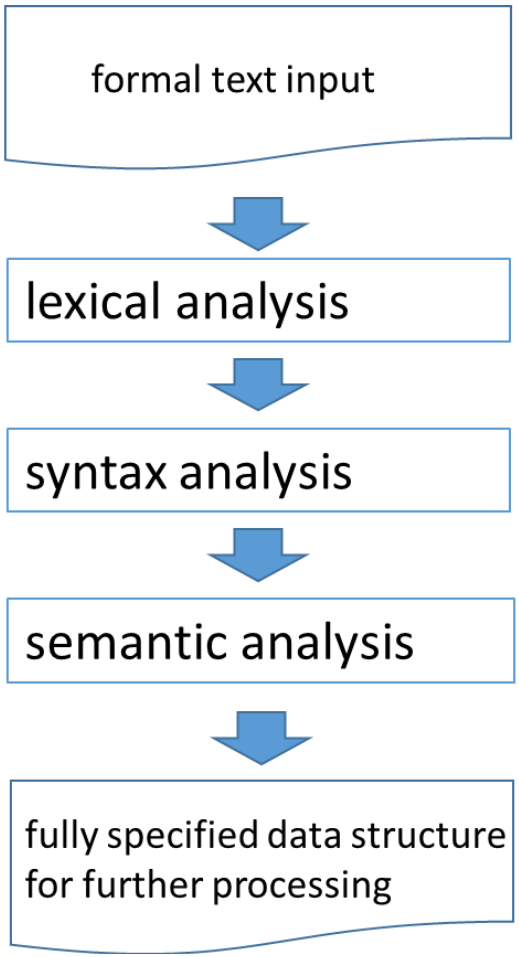
Algorithms

Results

Conclusion

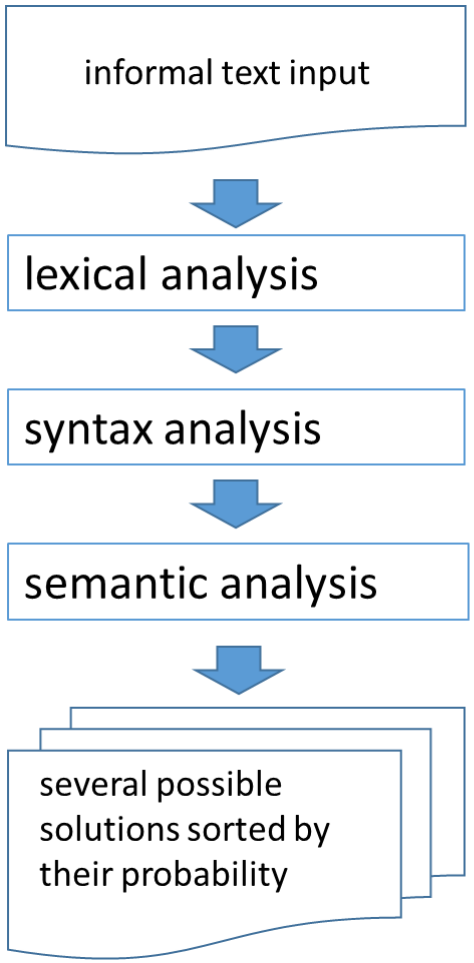
References

Traditional Parsing Approach:



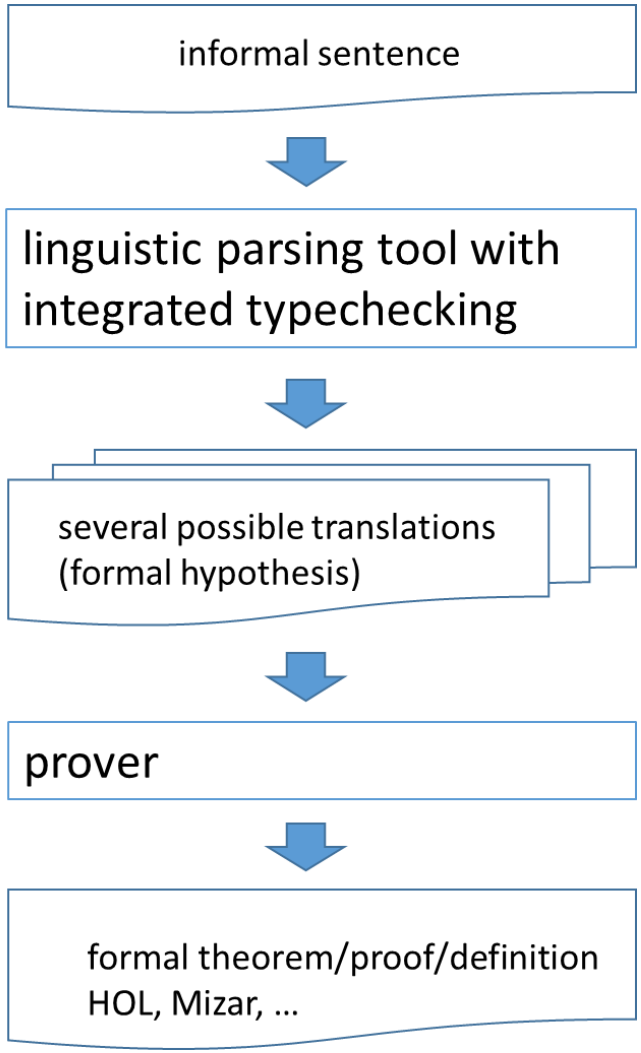
- a language is designed manually in such a way that:
  - lexical tokens can be fully specified by some *regular language*
  - syntax analyzer can be fully specified by some *unambiguous context free grammar* (typically by deterministic CFG)
  - semantic analyzer typically resolves types of symbols and subtrees in a parsing tree, checks semantic correctness of binders, ...

Linguistic Parsing Approach:



- all of these phases (or at least some of them) can be learned (instead of encoding them manually) from examples by machine learning
- syntax (and mostly even semantics) analysis can be done by ambiguous CFG with probabilities (PCFG) and lexical analysis (in case of English) is usually simple
- examples for learning have same (or similar) structure as parsing trees and they are called treebanks in this domain.
- rules and probabilities can be learned from treebanks
- CYK or Earley parser can be used for parsing such PCFG

Toolchain Overview:



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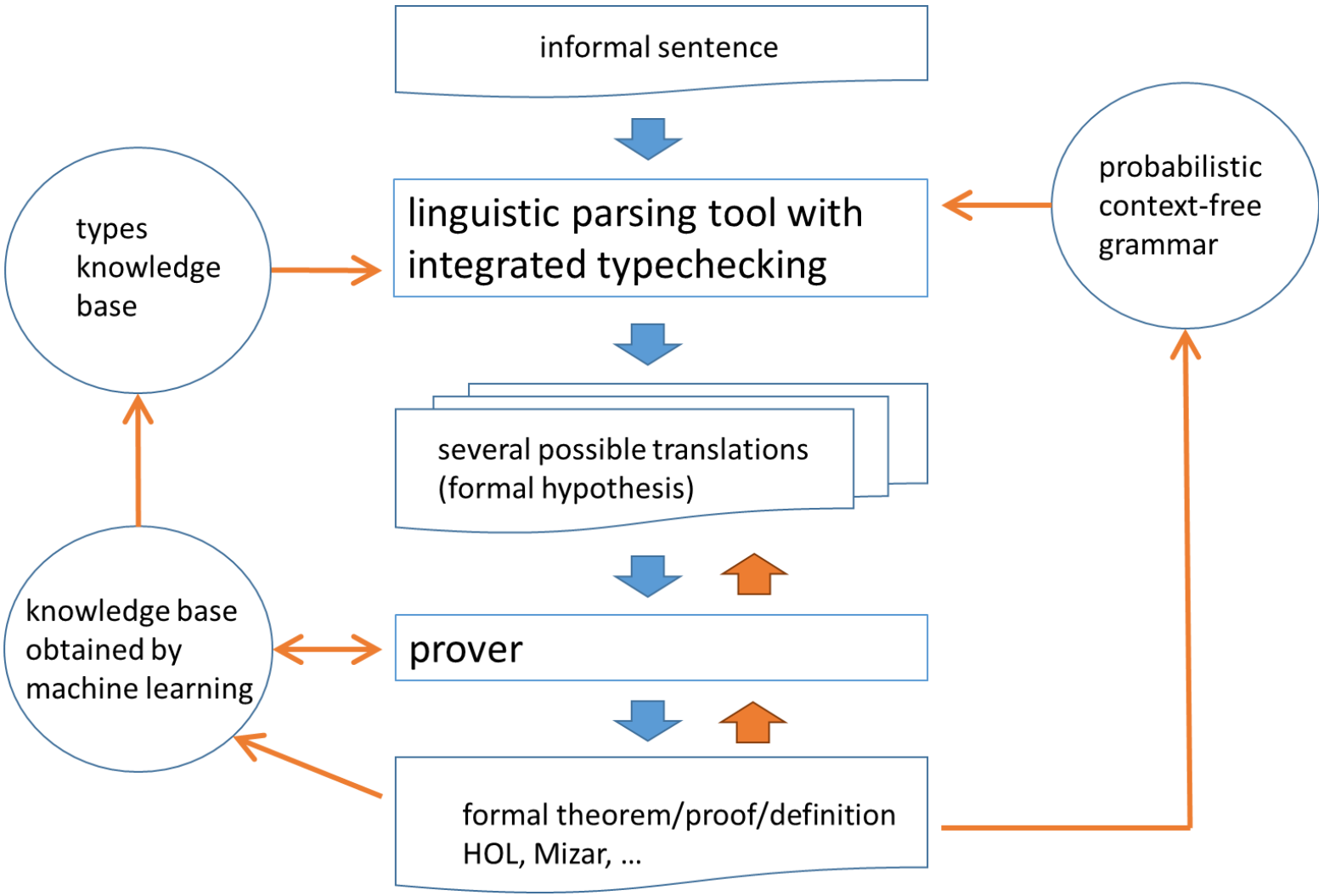

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- Mizar uses some predicates as types.
- "natural" or "non empty" or "Element of X"
- Saying t satisfies a type means a predicate like $P(t, s_1, \ldots, s_m)$ or $\neg P(t, s_1, \ldots, s_m)$ holds.
- The typing rules can essentially be classified as follows:
  - Types of functions f: If the arguments $x_1, \ldots, x_n$ satisfy certain types, then $f(x_1, \ldots, x_n)$ satisfies certain types.

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  - Redefinitions: A given function (or relation) applied to distinct variables satisfying certain types is equal (or equivalent to) a certain term (or proposition).

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    If A and B are sets, f is a function from A to B and x is an element of A, then $\mathrm{ap}'\, A\, B\, f\, x = \mathrm{ap}\, f\, x$.

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- For = four choices are `nr1_hidden`, `nr4_xboole_0`, `nr1_relat_1` and `nr1_complex1`.
- 8 possible readings given by the parser:
- $R(f(i,i),i)$ for one i, two f's and four R's.

Pattern Names vs. Constructors

- Names like `nk1_xxreal_0`, `nk1_xxreal_3` and `nr1_hidden` are "pattern names," not the real symbols which should occur in the real first-order terms and propositions.
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- Just drop the "n"? Not quite.
- `nk10_arytm_3` corresponds to k9_arytm_3.
- More importantly, the correspondence only holds when certain "pattern guards" hold.

Patterns

- Each pattern name n has a unique corresponding "pattern" of the form

$$\forall x_1, \cdots, x_k.\Phi \rightarrow n(x_j, \ldots, x_k) = f(t_1, \ldots, t_l)$$

or

$$\forall x_1, \cdots, x_k.\Phi \rightarrow n(x_j, \ldots, x_k) \equiv R(t_1, \ldots, t_l)$$

where the Φ is a set of typing conditions on the variables x_i.

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- On the other hand, Φ and the RHS of the conclusion are first-order formulas (or term, for $f(t_1, \ldots, t_l)$).
- The pattern for n gives necessary "pattern guards" (Φ) and indicates how to "elaborate" the use of a notation name n into a legitimate first-order term or formula.

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- Note: If $j > 1$, then $x_1, \ldots, x_{j-1}$ are "implicit arguments."

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- $R(f(i,i),i)$ for various pattern names i, f, R.
- For each choice, we can try to elaborate into a real first-order formula $R'(f'(i',i'),i')$.

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- IE, not all choices are well-typed (well-typed in the Mizar typing system).

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- Take f to be nk1_xxreal_3. The pattern rule for f:

If x and y are extended reals, then $f(x, y)$ is k1_xxreal_3(x, y).

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  If x and y are sets, then $R(x, y)$ is r1_hidden(x, y).

- The typing system knows every extended real is a set.
- Hence $R(f(i, i), i)$ elaborates to r1_hidden(k1_xxreal_3(i', i'), i').

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> *If x and y are natural numbers, then $f(x, y)$ is* k9_arytm_3(x, y).

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- Rule out an elaboration if too many pattern guards need to be assumed.

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  If x and y are natural numbers, then $f(x, y)$ is k9_arytm_3(x, y).

- extended real $\not\Rightarrow$ natural number.
- Stop the elaboration in this case and rule out this case.
- Slight problem: sometimes the checker is too weak, so we may assume some pattern guards and continue.
- Rule out an elaboration if too many pattern guards need to be assumed.
- In Example 1, only one parse is well-typed with no assumed guards.

Elaborating Example 1

- i is nk1_xxreal_0 which elaborates to k1_xxreal_0 (i').
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- Rule out an elaboration if too many pattern guards need to be assumed.
- In Example 1, only one parse is well-typed with no assumed guards.
- In the 7 other parses, at least 40% of checked pattern guards must be assumed.

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- The first dot depends on two arguments, the function and its argument:

```
definition
  let f be Function;
  let x be object ;
  func f . x -> set means :Def2: :: FUNCT_1:def 2
  [x,it] in f if x in dom f
  otherwise it = {} ;
  existence
  proof end;
  uniqueness by Def1;
  consistency ;
end;
```

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- In the MML, function application is defined twice, using an infix dot.
- The second dot is a redefinition depending on four arguments (the domain, codomain, function and the argument):

```
definition
  let C be non empty set ;
  let D be set ;
  let f be Function of C,D;
  let c be Element of C;
  :: original: .
  redefine func f . c -> Element of D;
  coherence
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- C, D are implicit in the pattern for the second:

If C is a nonempty set, D is a set, f is a function from C to D and c is in C, then nk3_funct_2(f, c) is k3_funct_2(C, D, f, c).

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- Using this type and the pattern for application we can recover the domain and codomain (the set of reals R) when elaborating both occurrences of application.
- Since 0 and k3_funct_2$(R, R,$ k16_sin_cos$, 0)$ can be checked to be elements of R, the parse can be fully elaborated as

$$\text{r1\_hidden}(\text{k3\_funct\_2}(R, R, \text{k16\_sin\_cos}, \text{k3\_funct\_2}(R, R, \text{k16\_sin\_cos}, 0)), 0)$$

Elaboration and Checking

- Two languages L_1 and L_2.
- Only L_1 has a sensible semantics. Each name in L_2 can be "elaborated" into an L_1 name.
- Elaborate proposition/term in language L_2 to one in language L_1.

$$s_2 \blacktriangleright s_1$$

- Doing this may require filling in implicit arguments.
- Result should be "well-typed" in the Mizar sense (type checking).
- The two processes interact since the way to obtain implicit arguments is by looking at types computed so far.

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- Each time a type is computed,
  - "widen" the type applying hierarchy rules and
  - "round up" cluster rules,
  - working modulo redefinition rules.

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- Otherwise check which pattern guards are known. Assume those that are unknown.
- Return $n(s_j, \ldots, s_k) \blacktriangleright \sigma(f(t_1, \ldots, t_l))$ and compute all types of $\sigma(f(t_1, \ldots, t_l))$.

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Elaborating Pattern Based Mizar Theorems

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- Is this fof constructor version equivalent to the constructor version of the corresponding theorem in the MPTP?
- Yes, and E can prove this equivalence in roughly 98% of cases.

Testing with the Parser

- The training data for the parser contains the pattern based versions as a parse tree (including presentation information such as literals like `for` and `implies`).
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- 75% of these 36K elaborated with assumed pattern guards
- 25% fail to fill in some implicit arguments
- When it works though, it gives a meaningful formula that can be sent to a theorem prover.

Planned Parser Tests

- We planned to do 100 fold crossvalidation on the training set.
- Current tests however show that the integration with the elaborator is not yet working well enough for the results to be meaningful.
- Without calling the elaborator, the parser can obtain the correct parse within the top 20 parses roughly 60% of the time.

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- With the elaborator, the correct parse is only found about 2% of the time.
- Here is one example that works with the elaborator but not without:

```
for seq being Real_Sequence holds ( seq is
  non-zero iff for n being Element of NAT
              holds seq .  n <> 0 )
```

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- MML in various representations.
- Probabilistic parser to learn from parse tree representations and then parse underlying text.
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- We have all the pieces to go from informal to formal, at least with Mizar data
- ...though some of the pieces are still being put together

Outline

Introduction

Parsing

Mizar Typing System

Examples

Algorithms

Results

Conclusion

References

References

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