

# Monotonic Abstraction Techniques: from Parametric to Software Model Checking

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# Aim of the talk

My talk will report recent experience concerning the development of an SMT-based model checker. Main features:

- declarative approach;
- use of decision procedures for combined theories;
- prominent role played by array fragments;
- quantifier handling through instantiation;
- quantifier handling through quantifier elimination;
- large expressivity;
- flexibility and possibility of integrating old and new techniques (acceleration, abstraction, invariant synthesis,...);
- large applications spectrum (distributed, timed, fault tolerant, but also sequential systems).

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# Verification of Parameterised Systems

- **Parameterised system** = bunch of concurrent processes (**topology** may vary, can be e.g., set-like, linear-like, tree-like, ring-like, ...)
- **Process** = instance of the same state-machine
- **Configuration** = state of a parameterised system
- **Transition** = either a process changing its locations/data or several processes simultaneously changing their respective locations/data (e.g., broadcast) [**interleaving semantics**]
- **CHALLENGE**: automatically verify a property regardless of the number of processes
- A state machine has **finitely many control locations** and can manipulate **finitely many variables over possibly unbounded domains**



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# Well-Structured Transition Systems

Seminal paper [ACJT - LICS96]

$$(S, \tau, \preceq)$$

- $S$ : set of states;
- $\tau = \{\rightarrow_{\lambda} \subseteq S \times S\}_{\lambda}$ : labelled directed graph;
- $\preceq$ : **well quasi ordering**<sup>1</sup> (wqo) on  $S$ ;
- each  $\tau_{\lambda}$  is **monotonic**:



<sup>1</sup>Reflexive, transitive binary relation that neither contains infinite strictly decreasing sequences nor infinite sequences of pairwise incomparable elements

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$$\begin{array}{ccc}
 s_1 & \preceq & s_2 \\
 \downarrow \lambda & & \downarrow \lambda \\
 s_3 & \preceq \exists & s_4
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# Well-Structured Transition Systems

- Set of **unsafe** states represented by an **upset**  $K$ :

$$s \in K \wedge s \preceq s' \rightarrow s' \in K$$

- Monotonicity implies that the **pre-image of an upset is still an upset**

$$Pre(\tau, K) := \{s \mid \exists \lambda \exists s' (s \xrightarrow{\lambda} s') \wedge s' \in K\}$$

- Since  $\preceq$  is a wqo, **upsets can be finitely represented by their *finitely many* minimal elements**

# Backward Reachability

Checking that a set  $K$  of **unsafe** states is (un-)reachable from a set  $I$  of **initial** states

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function BReach( $K$ )
   $i \leftarrow 0$ ;  $BR^0(\tau, K) \leftarrow K$ ;  $K^0 \leftarrow K$ 
  if  $BR^0(\tau, K) \cap I \neq \emptyset$  then return unsafe
  repeat
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- Since  $\preceq$  is a wqo, the algorithm terminates.
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# Monotonic Abstraction

But ... what to do if a transition  $\tau_\lambda$  is not monotonic?

We may have  $s \xrightarrow{\tau_\lambda} s'$  but  $\tilde{s} \xrightarrow{\tau_\lambda} s'$  for some  $\tilde{s} \preceq s$ .

In this case, *monotonic abstraction* allows  $\tau_\lambda$  to fire: the system may change its status from  $s$  to  $\tilde{s}$  to allow this.

Monotonic abstraction may introduce spurious runs (intuitively: runs in which some processes ‘crash and disappear’), but *if a safety certification is obtained for the abstract system, the certification holds for the original system too.*

Lot of success for the verification of safety properties of a variety of systems: broadcast protocols, cache coherence protocols, lossy channels systems, parameterized timed automata, etc.

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# Array-based Systems

**OUR GOAL:** to get a **declarative** formulation of all this and to obtain an **efficient backward reachability analysis** by using state-of-the-art **SMT solving** for both safety and fix-point checking.

By a *theory* we mean here a pair  $T = (\Sigma, \mathcal{C})$ , where  $\Sigma$  is a first-order signature and  $\mathcal{C}$  is a class of  $\Sigma$ -structures (called the models of  $T$ ). Satisfiability of at least quantifier-free formulae in  $\mathcal{C}$  should be decidable.

We need a theory  $T_I$  for describing processes and a theory  $T_E$  for data. We combine these two theories in a 3-sorted theory  $A_I^E$ .

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# Array-Based Systems

- the sort `INDEX` is constrained by  $T_I$ ;
- the sort `ELEM` is constrained by  $T_E$ ;
- the sort `ARRAY` represents arrays of `ELEM` defined on `INDEX`;
- the ‘read’ operation `_[_]` is added to  $\Sigma_I \cup \Sigma_E$ ;
- the class of models of  $A_I^E$  consists of the three-sorted structures whose reducts are models of  $T_I, T_E$  and the sort `ARRAY` is interpreted as the set of total functions from indexes to elements and the read operation is interpreted as function application

# Array-Based Systems

- An **array-based system** on  $A_I^E$  with array state variable  $a$  is the following pair of formulae:

$$\mathcal{S} = \langle I(a), \tau(a, a') \rangle.$$

- A **state** of an array-based system is an assignment to the variable  $a$  in a model of  $A_I^E$
- A **safety problem** for  $\mathcal{S}$  is the following: given a formula  $K(a)$ , is

$$I(a_0) \wedge \tau(a_0, a_1) \wedge \cdots \wedge \tau(a_{n-1}, a_n) \wedge K(a_n)$$

$A_I^E$ -satisfiable for some  $n$ ?

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# Revisiting Backward Reachability

Idea: recast symbolically the backward reachability algorithm

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Idea: recast symbolically the backward reachability algorithm

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  until  $A_f^E\text{-check}(\neg(BR^{i+1}(\tau, K) \rightarrow BR^i(\tau, K))) = \text{unsat}$ 
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But this is problematic... unless right formats for  $I, \tau, K$  are found!



# Format for initialization formulae

## Proposed format for $I$ : $\forall^I$ -formulae

$$\forall \underline{i} \phi(\underline{i}, a[\underline{i}])$$

where  $\underline{i}$  is a tuple of variables of sort INDEX and  $\phi$  is a quantifier-free  $\Sigma_I \cup \Sigma_E$ -formula<sup>2</sup>

For instance, the formula  $\forall i. a[i] = \text{idle}$  says that all processes are in state `idle`.

$\forall^I$ -formulae can also be used to express invariants

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<sup>2</sup>If  $\underline{i} = i_1, \dots, i_n$ , then  $a[\underline{i}]$  is the tuple of terms  $a[i_1], \dots, a[i_n]$  having sort ELEM.

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For instance, the formula  $\forall i. a[i] = \text{idle}$  says that all processes are in state `idle`.

$\forall^I$ -formulae can also be used to express invariants

---

<sup>2</sup>If  $\underline{i} = i_1, \dots, i_n$ , then  $a[\underline{i}]$  is the tuple of terms  $a[i_1], \dots, a[i_n]$  having sort ELEM.

# Format for unsafety problems formulae

**Proposed format for  $K$ :  $\exists'$ -formulae**

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where  $\underline{i}$  is a tuple of variables of sort INDEX and  $\phi$  is a quantifier-free  $\Sigma_I \cup \Sigma_E$ -formula.

For instance, the formula

$$\exists i_1 \exists i_2. (i_1 \neq i_2 \wedge \mathbf{a}[i_1] = \text{use} \wedge \mathbf{a}[i_2] = \text{use})$$

expresses that mutual exclusion is violated.

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## Format for transitions formulae

**Proposed format for  $\tau$ :** we use **disjunctions** of formulae of the kind

$$\exists \underline{i} \left( \phi_L(\underline{i}, \mathbf{a}[\underline{i}]) \wedge \mathbf{a}' = \lambda j F(\underline{i}, \mathbf{a}[\underline{i}], j, \mathbf{a}[j]) \right) \quad (1)$$

where  $F$  is a case-defined function (cases are described by quantifier-free formulae).

For instance, the formula

$$\exists i. \left( \mathbf{a}[i] = \text{use} \wedge \mathbf{a}' = \lambda j (\text{if } j = i \text{ then } \text{idle} \text{ else } \mathbf{a}[j]) \right)$$

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**Extended format for  $\tau$ :** results below apply also in case we use disjunctions of formulae in the more liberal format

$$\exists \underline{i} \exists \underline{e} \left( \phi_L(\underline{e}, \underline{i}, a[\underline{i}]) \wedge a' = \lambda j F(\underline{e}, \underline{i}, a[\underline{i}], j, a[j]) \right) \quad (2)$$

Existentially quantified data variables  $\exists \underline{e}$  are now allowed, but a *quantifier elimination* algorithm must be available for  $T_E$  - crucial for modeling **timed** systems.

An even more liberal format is obtained by replacing  $F$  with a **serial relation** - crucial for modeling nondeterminism in updates.



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## Universal quantifiers in guards

$$\exists \underline{i} \left( \phi_L(\underline{i}, \mathbf{a}[\underline{i}]) \wedge \forall j \psi(\underline{i}, j, \mathbf{a}[\underline{i}], \mathbf{a}[j]) \wedge \mathbf{a}' = \lambda j F(\underline{i}, \mathbf{a}[\underline{i}], j, \mathbf{a}[j]) \right) \quad (3)$$

can be eliminated by recasting **monotonic abstraction**.

In this declarative context, monotonic abstraction is simulated by **syntactic transformations**.

Roughly speaking, these syntactic transformations consist in adding a Boolean flag (crashed/active) and in relativizing quantifiers to active processes. [See our [JSAT 2013](#) paper for details]

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# Key points

- Closure: if  $H(a)$  is an  $\exists^!$ -formula, the formula  $Pre(\tau, H) := \exists a' (\tau(a, a') \wedge H(a'))$  is  $A_I^E$ -equivalent to an effectively computable  $\exists^!$ -formula: **true and easy!**
- Safety tests are effective: **generally true** (e.g. under mild assumptions on the shape of the initial formula).
- Fixpoint tests are effective: **true under certain assumptions** (but good - still incomplete - algorithms available in general).
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# The tool MCMT

- <http://users.mat.unimi.it/users/ghilardi/mcmt/>
- Obvious client-server architecture
- Client generates proof obligations ([satisfiability modulo theories problems](#))
- Server = state-of-the-art SMT solver (invoked via API)<sup>3</sup>
- Various heuristics implemented.
- Alternative recent implementation (on a parallel architecture, with additional sophisticated algorithms): CUBICLE  
<http://cubicle.lri.fr/>, by S. Conchon et al.

---

<sup>3</sup>[Yices](#) is the SMT-solver employed in MCMT.

# MCMT: mutual exclusion and cache coherence protocols

We first report benchmarks included in the distribution (in the best settings for the tool).<sup>4</sup>

## Mutual Exclusion

Problem	depth	#nodes	#deleted	#SMT calls	#inv.	time (sec)
Bakery_Lamport	4	7	1	222	7	0.03
Bakery_Bogus	8	90	14	1440	7	0.44
Distrib_Lamport	23	248	42	19622	7	27.87
Rickart_Agrawala	13	458	119	35241	0	148.24
Szymanski_atomic	9	21	9	3102	39	0.82

## Cache Coherence

Problem	depth	#nodes	#deleted	#SMT calls	#inv.	time (sec)
German	26	2121	255	117121	0	60.00
German_buggy	16	1631	203	40884	0	26.01
German_ca	9	13	0	216	0	0.02
Illinois	4	8	0	212	0	0.06

<sup>4</sup>The experiments were run on a laptop Intel(R) Core(TM) i3 CPU 2.27GHz with 4GB RAM running Linux

# MCMT: parametrized timed systems

- MCMT statistics

Problem	depth	#nodes	#SMT calls	time (sec)
Fischer_abd	14	111	5181	3.39
Fischer_sal	15	56	1186	0.34
Fischer_sal_buggy	6	16	307	0.08
Fischer_std	10	16	363	0.08
Fischer_upp	8	15	327	0.07
Lynch_mah	17	35	493	0.08
Lynch_full	25	1103	45554	37.56
CSMA	4	23	1363	0.61
CSMA_buggy	7	39	1778	0.90
tta	7	36	916	1.98
tta2	8	70	2017	14.00

- Uppaal timings (for increasing  $N$ )

Problem	$N = 2$	$N = 5$	$N = 10$
Fischer_sal	0.01	0.08	37392
Lynch_mah	0.00	0.05	44.34
CSMA	0.00	0.18	>10min
tta2	0.01	0.06	>10min

# MCMT: fault tolerant protocols

We analyzed a classical solution to the reliable broadcast problem (joint work with F. Alberti, E. Pagani, G. P. Rossi).



T. D. Chandra and S. Toueg.

Time and message efficient reliable broadcasts.

*In Proceedings of the 4th international workshop on Distributed Algorithms, 289–303, 1991.*

# MCMT: fault tolerant protocols

## Paper Overview

1. First Protocol for *Stopping-failure* model.  
⇒ This model is refined to *Send-Omission* model.
2. First Protocol is unsafe for this model.
3. Second modified version: still unsafe for *Send-Omission* model.
4. Third modified version: now safe for *Send-Omission* model!



# MCMT: fault tolerant protocols

**MCMT confirms all that!** In the last case, a little proof plan was needed (we asked the tool to first prove some lemmas suggested by us and then to attack the main task).

Problem	result	depth	#nodes	#deleted	#vars	#SMT calls	#inv.	time (sec)
Crash	SAFE	13	113	21	4	1731	0	0.75
Send_Omission (1)	UNSAFE	12	464	26	3	16253	0	14.16
Send_Omission (2)	UNSAFE	34	9679	770	6	1118959	0	30m 18.15s
Send_Omission (3)	SAFE	32	571	72	4	547054	94 (+7)	6m 57.19s

## Algorithm 1 Pseudo-code for Algorithms 1, 2, and 3

**Initialization:**

```

if ( $p$  is the sender)
  then  $estimate_p \leftarrow m$ ;  $coord\_id_p \leftarrow 0$ ;
  else  $estimate_p \leftarrow \perp$ ;  $coord\_id_p \leftarrow -1$ ;
 $state_p \leftarrow undecided$ ;

```

**End Initialization**

**for**  $c \leftarrow 1, 2, \dots$  **do** // Process  $c$  becomes coordinator for four rounds

**Round 1:**

```

All undecided processes  $p$  send request ( $estimate_p, coord\_id_p$ ) to  $c$ ;
if ( $c$  does not receive any request) then it skips rounds 2 to 4;
  else  $estimate_c \leftarrow estimate_p$  with largest  $coord\_id_p$ ;

```

**Round 2:**

```

 $c$  multicasts  $estimate_c$ ;
All undecided processes  $p$  that receive  $estimate_c$  do
   $estimate_p \leftarrow estimate_c$  and  $coord\_id_p \leftarrow c$ ;

```

**Round 3:**

```

All undecided processes  $p$  that do not receive  $estimate_c$  send(NACK) to  $c$ ;

```

**Round 4:**

```

if ( $c$  does not receive any NACK) then  $c$  multicasts Decide; else  $c$  HALTS;
All undecided processes  $p$  that receive Decide do
   $decision_p \leftarrow estimate_p$ ;
   $state_p \leftarrow DECIDED$ ;

```

**end for**

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# Monotonic Abstraction via Instantiation

Let us examine syntactic monotonic abstraction from another point of view. If we take an existential formula  $K$  and a transition  $\tau_h$  containing a universal guard, the preimage  $\text{Pre}(\tau_h, K)$  has the form

$$\exists \underline{i} \forall k \psi(\underline{i}, k, a[\underline{i}], a[k]), \quad (4)$$

where  $\psi$  is quantifier-free.

Instead of modifying syntactically  $\tau_h$  in order to eliminate from it the universal guard, we could over-approximate (4) via an existential formula at runtime (i.e. during backward search).

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# Monotonic Abstraction via Instantiation

The proposed overapproximation is the existential formula

$$\exists \underline{i} \bigwedge_t \psi(\underline{i}, t, a[\underline{i}], a[t]), \quad (5)$$

varying  $t$  among a set of terms  $X$ . We may call (5) a *syntactic monotonic abstraction of the formula* (4) (notice that this notion is relative to  $X$ ).

If one take the obvious choice  $X := \underline{i}$ , we do not get *in the end* anything different from syntactic monotonic abstraction applied to transitions. But the situation becomes different (we have more flexibility), when there is some arithmetics on indexes.

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# Array Acceleration

This observation can be exploited in software model checking when dealing with programs for arrays of unbounded length. We show the technique by an example.

The following ‘initialize-and-test’ simple example is considered problematic for CEGAR techniques:

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for(I=0; I!= a_length; I++) a[I]=0;
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Indeed backward search trivially diverges here:

$$p = 2 \wedge J \neq a\_length \wedge a[J] \neq 0$$

$$p = 2 \wedge J + 1 \neq a\_length \wedge a[J + 1] \neq 0 \wedge a[J] = 0$$

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$$p = 2 \wedge J + n \neq a\_length \wedge a[J + n] \neq 0 \wedge \bigwedge_{k=J}^{J+n-1} a[k] = 0$$

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To stop divergence, we need to re-introduce quantifiers. One possible solution is to summarize the effect of  $n$  executions of a loop into a single transition, representing transitive closure. This technique is known as *acceleration* in model-checking and has been extensively investigated for fragments of Presburger arithmetic.

In the example above, we can accelerate the two loops, resulting in

$$\begin{aligned} & \exists n > 0 \left( \begin{array}{l} p = 1 \wedge \forall k (I \leq k < I + n \rightarrow k \neq a\_length) \wedge p' = 1 \wedge \\ I' = I + n \wedge J' = J \wedge a' = wr(a, [I, I + n - 1], 0) \end{array} \right); \\ & \exists n > 0 \left( \begin{array}{l} p = 2 \wedge \forall k (J \leq k < J + n \rightarrow k \neq a\_length \wedge a[k] = 0) \\ \wedge p' = 2 \wedge I' = I \wedge J' = J + n \wedge a' = a \end{array} \right). \end{aligned}$$

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# Monotonic Abstraction for Arrays

There are however remarkable differences in the use of abstraction here wrt the distributed case.

- Monotonic abstraction here is just an abstraction technique among many others (we loose intuitive justifications in terms of crash failures).
- Monotonic abstraction can produce spurious traces, but here we can *ignore* such spurious traces: no refinement is needed, one simply drops unsafe traces containing accelerations (if the system is unsafe, unsafety should be discovered without acceleration!)
- Our monotonic abstraction is purely syntactic, hence it can be used *in combination with other abstraction techniques* (in MCMT it is combined with predicate abstraction via interpolants).



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# Monotonic Abstraction for Arrays

The combination with monotonic abstraction with other abstraction is quite powerful: typically, when there are nested loops, monotonic abstraction takes care of inner loops, thus leaving predicate abstraction to care about outer loops only.

Very often, array accelerated transitions gives formulae in an  $\exists^*\forall^*$ -fragment which is decidable modulo array axioms (Bradley fragment, our flat fragment [TACAS 14], ...). In these cases, when the control flow graph is flat, safety is decidable and it is convenient not to use any abstraction at all.

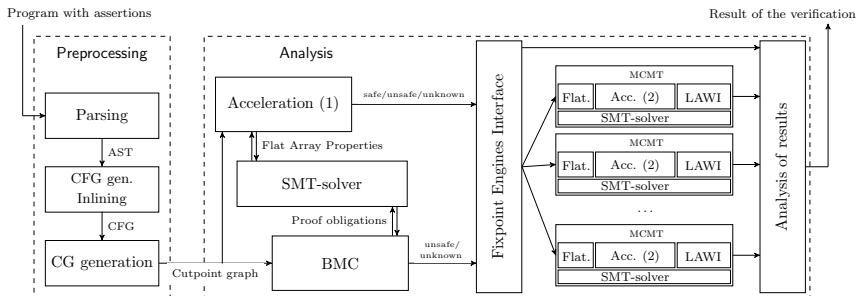
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# The BOOSTER Tool

An acceleration-based software model-checker



F. Alberti, S. Ghilardi, and N. Sharygina.

Booster: an acceleration-based verification framework for array programs

In *ATVA*, Springer, 2014. To appear.

# BOOSTER: Experiments

FILENAME	STATUS	ACC+ABS	ABS	ACC
data_structures/set_multi_proc.c	SAFE	1.600	TO	TO
data_structures/set_multi_proc_trivial.c	SAFE	0.208	0.208	0.314
data_structures/set_multi_proc_unsafe.c	UNSAFE	1.946	1.257	2.102
sanfoundry/06.c	SAFE	0.016	TO	0.016
sanfoundry/07.c	SAFE	4.623	TO	TO
sanfoundry/08.c	SAFE	2.926	TO	TO
sanfoundry/09.c	SAFE	8.447	TO	TO
sanfoundry/10.c	SAFE	0.157	TO	TO
sanfoundry/24.c	SAFE	0.101	0.071	0.085
sanfoundry/27.c	SAFE	0.066	0.076	108.724
sanfoundry/28.c	SAFE	0.676	0.151	63.932
sanfoundry/39.c	SAFE	1.832	TO	TO
sorting/bubblesort.c	SAFE	0.233	0.107	0.407
sorting/bubblesort_unsafe.c	UNSAFE	0.090	0.090	0.135
sorting/selectionsort.c	SAFE	85.326	TO	TO
sorting/selectionsort_unsafe.c	UNSAFE	1.500	1.658	1.629
standard/allDiff_safe.c	SAFE	0.010	0.044	0.010
standard/allDiff_unsafe.c	UNSAFE	0.007	0.036	0.006
svcomp/loops/array_false-unreach-label.c	UNSAFE	0.135	0.039	0.094
svcomp/loops/array_true-unreach-label.c	SAFE	0.169	0.057	TO
svcomp/loops/compact_false-unreach-label.c	UNSAFE	0.010	0.051	0.010
svcomp/loops/heavy_false-unreach-label.c	SAFE	0.363	0.277	TO
svcomp/loops/heavy_true-unreach-label.c	UNSAFE	0.296	0.217	0.393
svcomp/loops/linear_search_false-unreach-label.c	UNSAFE	0.154	0.053	0.062
svcomp/loops/linear_search_true-unreach-label.c	SAFE	0.016	0.101	TO
svcomp/loops/nec11_false-unreach-label.c	UNSAFE	0.053	0.040	0.75
svcomp/loops/nec40_true-unreach-label.c	SAFE	0.010	0.607	0.16
svcomp/loops/string_true-unreach-label.c	SAFE	0.860	0.781	1.04
svcomp/loops/sum_array_false-unreach-label.c	UNSAFE	0.068	0.059	0.104
svcomp/loops/sum_array_true-unreach-label.c	SAFE	0.070	0.080	TO

# BOOSTER: Comparisons (?)

BENCHMARK	COMPASS	Z3 HORN	ARMC	DUALITY	BOOSTER
init	0.01	0.06	0.15	0.72	0.01
init_non_constant	0.02	0.08	0.48	6.60	0.01
init_partial	0.01	0.03	0.14	2.60	0.01
init_partial_buggy	0.02	0.01	0.07	0.03	0.01
init_even	0.04	TO	?	TO	0.02
init_even_buggy	0.04	NA	NA	NA	0.01
copy	0.01	0.04	0.20	1.40	0.01
copy_partial	0.01	0.04	0.21	1.80	0.01
copy_odd	0.04	TO	?	4.50	TO
copy_odd_buggy	0.05	NA	NA	NA	0.07
reverse	0.03	0.12	2.28	8.50	0.02
reverse_buggy	0.04	0.01	0.08	0.03	0.01
swap	0.12	0.41	3.0	40.60	0.12
swap_buggy	0.11	NA	NA	NA	0.03
double_swap	0.16	1.37	4.4	TO	0.34
check_strcpy	0.07	0.05	0.15	0.62	0.02
check_memcpy	0.04	0.04	0.20	16.30	0.02
find	0.02	0.01	0.08	0.38	0.26
find_first_nonnull	0.02	0.01	0.08	0.39	0.09
array_append	0.02	0.04	1.76	1.50	0.02
merge_interleave	0.09	0.04	?	1.50	0.15
merge_interleave_buggy	0.11	NA	NA	NA	0.01



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- This syntactic reformulation can be **combined with acceleration** in other applications domains (eg model checking sequential array programs).
- The resulting technique turns out to be **simple, easily implementable and quite effective**.
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