A Concurrency Problem with Exponential DPLL($\mathcal{T}$) Proofs

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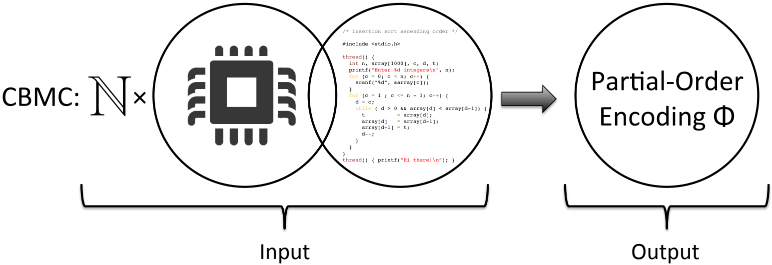
Outline

SAT/SMT-based Verification Techniques

SAT/SMT solvers are highly optimized decision procedures:

$$\phi \rightarrow \square \begin{array}{l} \nearrow \text{SAT} \\ \searrow \text{UNSAT} \end{array}$$

Using these, state-of-the-art symbolic bounded model checker such as CBMC can find concurrency bugs in Apache, PostgreSQL and .

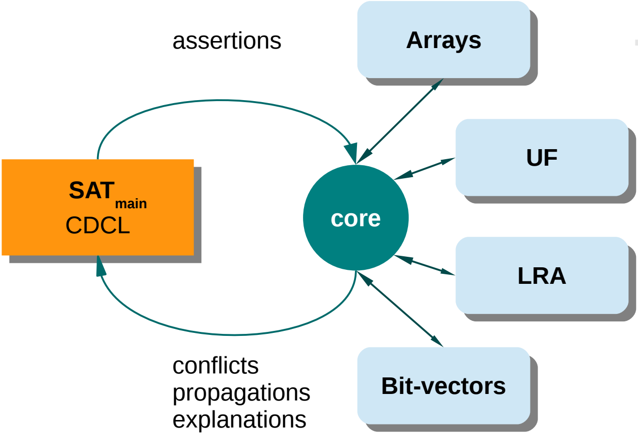


The Problem

Can we pinpoint the challenges (if any) symbolic partial-order encodings of concurrency pose for SAT/SMT solvers?

If yes, what insights can we gain?

SMT Solvers built on DPLL($\mathcal{T}$)



Fixed-Alphabet DPLL($\mathcal{T}$) Proofs

The SMT solvers in our experiments are built on the DPLL($\mathcal{T}$) framework. A simplified form of DPLL($\mathcal{T}$) with only two rules:

- Propositional resolution (Res);
- Learning $\mathcal{T}$-valid clauses over the literals of a fixed alphabet of $\mathcal{T}$-atoms ($\mathcal{T}$-learn).

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Example: $(x < y \vee x = y) \wedge y < x$ where $x, y \in \mathbb{Z}$. This formula is $\mathcal{T}$-unsatisfiable where $\mathcal{T}$ is `QF_LIA`.

Fixed-Alphabet DPLL($\mathcal{T}$) Proofs

Example: $(\overbrace{x < y}^{A} \vee \overbrace{x = y}^{B}) \wedge \overbrace{y < x}^{C}$

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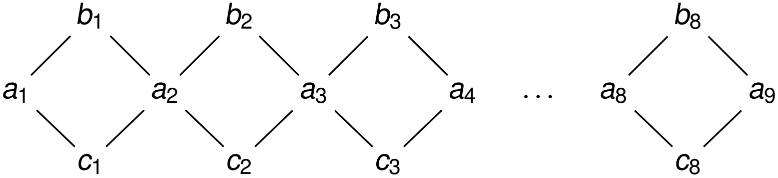
$$\rightsquigarrow \bot \qquad \text{(RES)}$$

Known Challenges for SMT Solvers

$$\phi_\diamond \triangleq a_1 \neq a_9 \wedge \bigwedge_{i=1}^{8} (a_i = b_i \wedge b_i = a_{i+1}) \vee (a_i = c_i \wedge c_i = a_{i+1})$$



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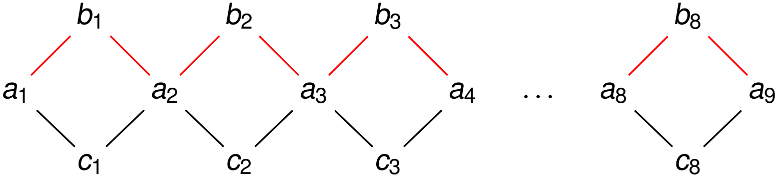
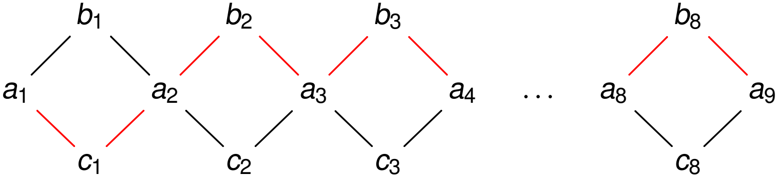


Known Challenges for SMT Solvers

$$\phi_\diamond \triangleq a_1 \neq a_9 \land \bigwedge_{i=1}^{8} (a_i = b_i \land b_i = a_{i+1}) \lor (a_i = c_i \land c_i = a_{i+1})$$



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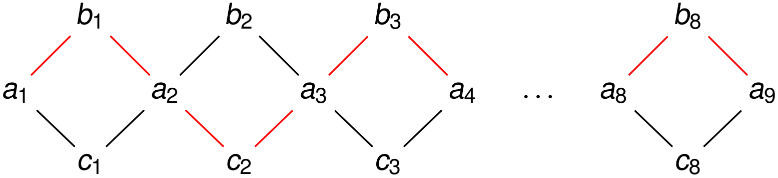

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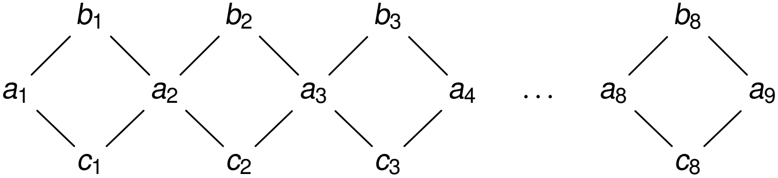
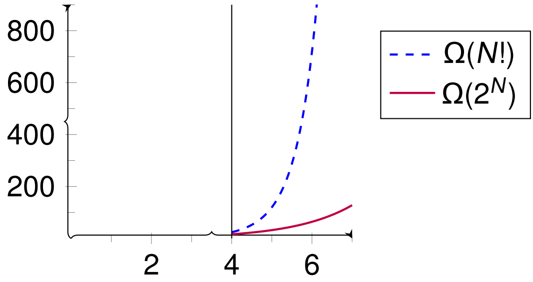


- Note that $\phi_\diamond$ is unsatisfiable because $a_i = a_{i+1}$ for all $1 \leq i \leq 8$.
- But DPLL($\mathcal{T}$) cannot learn these equalities and enumerates 2^8 theory conflicts instead, the infamous *diamonds problem*.[1]

[1] Bjørner, N., Dutertre, B., de Moura, L.: *Accelerating Lemma Learning using Joins - DPLL(Join)*. In: LPAR (2008)

Contributions

- A general framework for establishing lower bounds on the number of $\mathcal{T}$-conflicts in the fixed-alphabet DPLL($\mathcal{T}$) calculus;
- Proof of factorial lower bound proof complexity for two state-of-the-art symbolic partial-order encodings of a simple, yet challenging concurrency problem;
- Experiments that confirm our theoretical lower bound.



DPLL($\mathcal{T}$) Lower Bound Proof Complexity

Informally, *non-interfering set of critical assignments* are propositionally satisfying assignments that contain minimal, disjoint $\mathcal{T}$-conflicts.

Theorem

Let ϕ be an unsatisfiable $\mathcal{T}$-formula, and Q be a non-interfering set of critical assignments for ϕ. Every Fixed-Alphabet-DPLL($\mathcal{T}$) proof that ϕ is UNSAT contains at least |Q| applications of $\mathcal{T}$-LEARN.

This theorem is a theoretical tool for proving lower bound proof complexity results in the DPLL($\mathcal{T}$) framework (as exhibited next).

The Concurrency Problem

The value at memory location x is initialized to zero, i.e. $[x] = 0$.

$$
\begin{array}{l|l|c|l}
\text{Thread } \mathtt{T}_0 & \text{Thread } \mathtt{T}_1 & & \text{Thread } \mathtt{T}_N \\
\hline
\textbf{local } v_0 := [x] & \textbf{local } v_1 := [x] & \ldots & \textbf{local } v_N := [x] \\
\textbf{assert}(v_0 \leq N) & [x] := v_1 + 1 & & [x] := v_N + 1
\end{array}
$$

$$
\begin{array}{c|c|c|c}
\text{Thread } \mathtt{T}_0 & \text{Thread } \mathtt{T}_1 & & \text{Thread } \mathtt{T}_N \\
\hline
r_0 & r_1 & \ldots & r_N \\
 & w_1 & & w_N
\end{array}
$$

For $N = 2$, if restricted to $\mathtt{T}_1 \parallel \mathtt{T}_2$, we get the following interleavings:

(1) $r_1 ; w_1 ; r_2 ; w_2$ (2) $r_1 ; r_2 ; w_1 ; w_2$ (3) $r_1 ; r_2 ; w_2 ; w_1$
(4) $r_2 ; r_1 ; w_1 ; w_2$ (5) $r_2 ; r_1 ; w_2 ; w_1$ (6) $r_2 ; w_2 ; r_1 ; w_1$.

In general, we get $(2N + 1)! \div 2^N$ interleavings for $\mathtt{T}_0 \parallel \mathtt{T}_1 \parallel \ldots \parallel \mathtt{T}_N$.

Concurrency Problem Encoding

We define the following preserved-program order (PPO) for the problem challenge $\mathtt{T}_0 \parallel \mathtt{T}_1 \parallel \ldots \parallel \mathtt{T}_N$:

$$\begin{tikzcd}
 & w_{init} \arrow[dl] \arrow[d] \arrow[drr] & & \\
r_0 & r_1 \arrow[d] & \dots & r_N \arrow[d] \\
 & w_1 & \dots & w_N
\end{tikzcd}$$

Let $\mathrm{R} \triangleq \{r_0, \ldots, r_N\}$ and $\mathrm{W} \triangleq \{w_{init}, w_0, \ldots, w_N\}$.

Our partial-order encodings are parameterized by three theories:

- $\mathcal{T}_C$: clock constraints, e.g. $\mathrm{c}_r \prec \mathrm{c}_w$ for $r \in \mathrm{R}$ and $w \in \mathrm{W}$,
- $\mathcal{T}_S$: selection constraints, e.g. $\mathrm{s}_r = \mathrm{s}_w$ for $r \in \mathrm{R}$ and $w \in \mathrm{W}$,
- $\mathcal{T}_V$: read constraints, e.g. rv_r is a unique $\mathcal{T}_V$-variable for $r \in \mathrm{R}$.

Cubic-size Encoding of Concurrency Problem

Let ϕ^3 be the $O(N^3)$ partial-order encoding of $\mathtt{T}_0 \parallel \mathtt{T}_1 \parallel \ldots \parallel \mathtt{T}_N$:

$$\underbrace{c_{w_{init}} \prec c_{r_{assert}} \wedge \bigwedge_{i=1\ldots N} c_{w_{init}} \prec c_{r_i} \prec c_{w_i}}_{\textbf{PPO}} \wedge \underbrace{\bigwedge_{w,w' \in W, w \neq w'} (c_w \prec c_{w'} \vee c_{w'} \prec c_w) \wedge s_w \neq s_{w'}}_{\textbf{WW}[x]} \wedge$$

$$\underbrace{\bigwedge_{w \in W, r \in R} (c_w \prec c_r \vee c_r \prec c_w)}_{\textbf{RW}[x]} \wedge \underbrace{\bigwedge_{r \in R} \left(\bigvee_{w \in W} s_w = s_r \right)}_{\textbf{RF}_{\textbf{TO}}[x]} \wedge \underbrace{rv_{r_{assert}} > N}_{\textbf{assert}(v_0 \leq N)} \wedge$$

$$\underbrace{\bigwedge_{w \in W, r \in R} (s_w = s_r) \Rightarrow c_w \prec c_r}_{\textbf{RF}^3[x]} \wedge \underbrace{\bigwedge_{r \in R} (s_{w_{init}} = s_r) \Rightarrow 0 = rv_r}_{\textbf{RF}^3[x]} \wedge \underbrace{\bigwedge_{i=1\ldots N, r \in R} (s_{w_i} = s_r) \Rightarrow rv_{r_i} + 1 = rv_r}_{\textbf{RF}^3[x]}$$

$$\underbrace{\bigwedge_{w,w' \in W, r \in R} (s_w = s_r \wedge c_w \prec c_{w'}) \Rightarrow c_r \prec c_{w'}}_{\textbf{FR}[x]}$$

Factorial Lower Bound DPLL($\mathcal{T}$) Proof Complexity

Theorem (Lower Bound for Cubic Partial-Order Encoding)
All Fixed-Alphabet-DPLL($\mathcal{T}$) proofs for the problem challenge encoded with ϕ^3 contain at least $N!$ applications of $\mathcal{T}$-LEARN.

We also have a quadratic-size partial-order encoding.[2]

This asymptotically smaller encoding has also at least factorial-sized DPLL($\mathcal{T}$) proofs for our challenge problem!

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[2] *Horn and Kroening. On Partial Order Semantics for SAT/SMT-based Symbolic Encodings of Weak Memory Concurrency*. In: FORTE'15.

Experiments with Two Partial-Order Encodings

$\mathcal{E}^3$ and $\mathcal{E}^2$ are partial-order encodings of asymptotically different size, parameterized by three theories $\mathcal{T}_C$, $\mathcal{T}_S$ and $\mathcal{T}_V$.

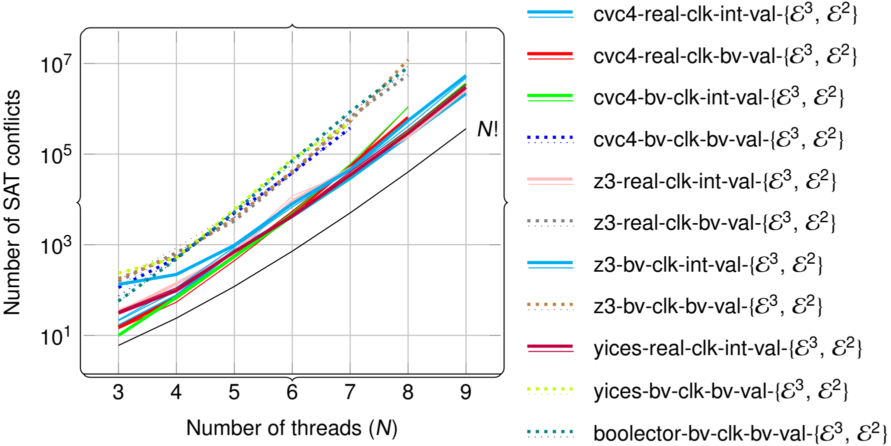
We instantiate $\langle \mathcal{T}_C, \mathcal{T}_S, \mathcal{T}_V \rangle$ to four configurations:

1. "real-clk-int-val": $\mathcal{T}_C = \mathcal{T}_S = \mathcal{T}_{\mathbb{R}}$ and $\mathcal{T}_V = \mathcal{T}_{\mathbb{Z}}$
2. "bv-clk-int-val": $\mathcal{T}_C = \mathcal{T}_S = \mathcal{T}_{\mathbb{BV}}$ and $\mathcal{T}_V = \mathcal{T}_{\mathbb{Z}}$
3. "real-clk-bv-val": $\mathcal{T}_C = \mathcal{T}_S = \mathcal{T}_{\mathbb{R}}$ and $\mathcal{T}_V = \mathcal{T}_{\mathbb{BV}}$
4. "bv-clk-bv-val": $\mathcal{T}_C = \mathcal{T}_S = \mathcal{T}_{\mathbb{BV}}$ and $\mathcal{T}_V = \mathcal{T}_{\mathbb{BV}}$

We use the following SMT solvers: Boolector, CVC4, Yices2, Z3.

Example: "z3-bv-clk-int-val-$\mathcal{E}^2$" denotes experiments with the $O(N^2)$ encoding using Z3 where $\mathcal{T}_C = \mathcal{T}_S = \mathcal{T}_{\mathbb{BV}}$ and $\mathcal{T}_V = \mathcal{T}_{\mathbb{Z}}$. We have a total of 56 SMT-LIB benchmarks. Timeout is 1 hour.

Experimental Results



Factorial growth of conflicts in `fkp2013-unsat` benchmark.

Concluding Remarks

- We have studied a simple, yet challenging concurrency problem.
- Our experiments provide an important diagnostic practice in the development of SMT encodings.
- The proofs we have manually inspected in CVC4 pinpoint value constraints as culprits.
- This way, our experiments can guide research into improving the performance of SMT solvers on such benchmarks.

The results of our work will shortly be published in SMT'15.

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Thank you!

SC-relaxed Consistency Encoding

Let E be the set of events, $\ll$ be the PPO, $val : \mathrm{E} \to \mathcal{T}_V$-terms, $guard : \mathrm{E} \to \mathcal{T}_V$-formulas and L be the set of memory locations.

$$\mathbf{PPO} \triangleq \bigwedge \{(guard(e) \wedge guard(e')) \Rightarrow (\mathsf{c}_e \prec \mathsf{c}_{e'}) \mid e, e' \in \mathrm{E} : e \ll e'\}$$

$$\mathbf{WW}[x] \triangleq \bigwedge \{(\mathsf{c}_w \prec \mathsf{c}_{w'} \vee \mathsf{c}_{w'} \prec \mathsf{c}_w) \wedge \mathsf{s}_w \neq \mathsf{s}_{w'} \mid w, w' \in \mathrm{W}_x \wedge w \neq w'\}$$

$$\mathbf{RW}[x] \triangleq \bigwedge \{\mathsf{c}_w \prec \mathsf{c}_r \vee \mathsf{c}_r \prec \mathsf{c}_w \mid w \in \mathrm{W}_x \wedge r \in \mathrm{R}_x\}$$

$$\mathbf{RF_{TO}}[x] \triangleq \bigwedge \{guard(r) \Rightarrow \bigvee \{\mathsf{s}_w = \mathsf{s}_r \mid w \in \mathrm{W}_x\} \mid r \in \mathrm{R}_x\}$$

$$\mathbf{RF}^3[x] \triangleq \bigwedge \{(\mathsf{s}_w = \mathsf{s}_r) \Rightarrow (guard(w) \wedge val(w) = \mathsf{rv}_r \wedge \mathsf{c}_w \prec \mathsf{c}_r) \mid r \in \mathrm{R}_x \wedge w \in \mathrm{W}_x\}$$

$$\mathbf{FR}[x] \triangleq \bigwedge \{(\mathsf{s}_w = \mathsf{s}_r \wedge \mathsf{c}_w \prec \mathsf{c}_{w'} \wedge guard(w')) \Rightarrow (\mathsf{c}_r \prec \mathsf{c}_{w'}) \mid w, w' \in \mathrm{W}_x \wedge r \in \mathrm{R}_x\}$$

$$\mathcal{E}^3 \triangleq \bigwedge \left\{\mathbf{RF_{TO}}[x] \wedge \mathbf{RF}^3[x] \wedge \mathbf{FR}[x] \wedge \mathbf{WW}[x] \wedge \mathbf{RW}[x] \mid x \in L\right\} \wedge \mathbf{PPO}$$

$$\mathbf{RF}^2[x] \triangleq \bigwedge \{(\mathsf{s}_w = \mathsf{s}_r) \Rightarrow (\mathsf{c}_w = \mathsf{sup}_r \wedge guard(w) \wedge val(w) = \mathsf{rv}_r \wedge \mathsf{c}_w \prec \mathsf{c}_r) \mid r \in \mathrm{R}_x \wedge w \in \mathrm{W}_x\}$$

$$\mathbf{SUP}[x] \triangleq \bigwedge \{(\mathsf{c}_w \preceq \mathsf{c}_r \wedge guard(w)) \Rightarrow (\mathsf{c}_w \preceq \mathsf{sup}_r) \mid r \in \mathrm{R}_x \wedge w \in \mathrm{W}_x\}$$

$$\mathcal{E}^2 \triangleq \bigwedge \left\{\mathbf{RF_{TO}}[x] \wedge \mathbf{RF}^2[x] \wedge \mathbf{SUP}[x] \wedge \mathbf{WW}[x] \wedge \mathbf{RW}[x] \mid x \in L\right\} \wedge \mathbf{PPO}$$