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Maintaining Latest Information Beyond Channel Bounds

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Distributed Systems

Finite set of processes
Communicating via reliable FIFO message passing
multiple channels between processes
Remote control light
Obey the latest order
Gossip

• Cooperate so that every process maintains latest information about every other process
• When receiving a message, a process needs to identify which is more recent:
  • the information it has,
  • the information transmitted by the sender
How to maintain the latest information?
How to maintain the latest information?
Why
How to maintain the latest information using only finite set of messages?

- Finite communication complexity
- Formal Methods
  - Model checking
- Distributed Synthesis
- Global Snapshots
  - Causal Ordering
- Bounded implementations of replicated data-types
- Local testing
How to maintain the latest information using only finite set of messages?

need to reuse tags

No natural ordering between the tags
How to maintain the latest information using only finite set of messages?

We use some secondary knowledge

No natural ordering between the tags
How to maintain the latest information using only finite set of messages?

- Is it even possible?
  - Synchronous communication [Zielonka87]
- At least in some cases?
  - Bounded channels [Mukund et al.03]
- Beyond Bounded channels?
How to maintain the latest information using only finite set of messages?

When is a color not needed any more?

I can reuse a color when I know that the tagged message has been received.

I know that everyone knows that the tagged message has been received.

Colors are not freed in the order they were used.

k-Bounded channels permit finite time-stamping.

Secondary knowledge requires $k^2$ colors.

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Secondary knowledge requires $k$ colors.

Secondary knowledge shows a bound, and using a round-robin does not work.
How to maintain the latest information using only finite set of messages?

- k-Bounded channels permit finite time-stamping
- Are channel bounds necessary for finite time-stamping?
- Equivalent writes
- Not simply stuttering
- Important writes
- Are existential channel bounds necessary?
Important writes

How to maintain the latest information using only finite set of messages?

Are existential channel bounds necessary?

Equivalent writes

Important writes
We need some bound:
Primary information

- Pending writes
- Equivalent writes
- Primary writes
We need some bound: Primary information

We solve the gossip problem for primary bounded
How do we maintain the primary?

Keeping primary alone is not enough

Need secondary knowledge

What is secondary knowledge?
Secondary = Primary of Primary
Secondary = Primary of Primary

When can I reuse a color?
- When it is not in the secondary
- \( \sim k^2 \) colors
- \( \sim \) size of secondary

Can we maintain secondary?
- YES, WE CAN!
Gossip: more precisely

- Message passing automaton (MPA or CFM)

Gossip = (Locs, (Trans_p)_{p \in \text{Procs}})

Run: \( \rho : \text{Events} \rightarrow \text{Locs} \)
Known and Latest

Known: Locs $\rightarrow 2^{\text{Procs}}$

Known($\ell''$) = \{p_2, p_3, \ldots, p_6\}

Latest: Locs$^2$ $\rightarrow 2^{\text{Procs}}$

Latest($\ell, \ell'$) = \{p_3, p_5, p_6\}
Colors and time-stamps

\[ \chi(g) = \min(\mathbb{N} \setminus \chi(\text{Sec}(\downarrow g) \cap \text{Send}(d))) \]

\[ h(g) = (d, \chi(g)) \]

\[ K^1(g) = \{ h(e) \mid e \in \text{Prim}(\downarrow g) \} \]
Locations of Gossip

\[ K^2(g) = (\text{pid}(g), d, c, K^1(g), (K^1(e))_{e \in \text{Prim}(g)}) \]

\[ \ell = (p, d, c, P, (S_{\gamma})_{\gamma \in P}) \]
Locations of Gossip

\[ K^2(g) = (\text{pid}(g), d, c, K^1(g), (K^1(e))_{e \in K^1(g)}) \]

\[ \ell = (p, d, c, P, (S_\gamma)_{\gamma \in P}) \]
Known

\[ \ell = (p, d, c, P, (S_{\gamma})_{\gamma \in P}) \]

\[ \text{pid}(\downarrow g) = \{ \text{pid}(g) \} \cup \text{pid}(\text{Prim}(\downarrow g)) \]

\[ \text{Known}(\ell) = \{ p \} \cup \text{pid}(P) \]
Maintaining $K^2$

write event case 1

p \rightarrow e \rightarrow g

$3 \xrightarrow{d} e \xrightarrow{l} g \xrightarrow{d} l' = l$

Equivalent writes: no changes
Maintaining $K^2$

New channel: requires an available color

\[ \ell = (p, d, c, P, (S_\gamma)_{\gamma \in P}) \]
\[ \ell' = (p, d', c', P' = P \cup \{(d', c')\}, (S'_\gamma)_{\gamma \in P'}) \]
Maintaining $K^2$

$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$

$\ell' = (p', d', c', P', (S'_\gamma)_{\gamma \in P'})$

$f < c$ iff

$(d', c') \in P \land \exists (d'', c'') \in P \setminus P' \ (d'' \neq d' \land W(d'') = W(d'))$
Maintaining $K^2$

\[ \ell = (p, d, c, P, (S_{\gamma})_{\gamma \in P}) \]
\[ \ell' = (p', d', c', P', (S_{\gamma}')_{\gamma \in P'}) \]

\[ f < e \text{ iff } \]
\[ (d', c') \in P \land \exists (d'', c'') \in P \setminus P' (d'' \neq d' \land W(d'') = W(d')) \]
Maintaining \( K^2 \)

**Read event case 1**

\[
\text{Latest}(\ell, \ell') = \text{Known}(\ell)
\]

\[
\ell = (p, d, c, P, (S_{\gamma})_{\gamma \in P})
\]

\[
\ell' = (p', d', c', P', (S'_{\gamma})_{\gamma \in P'})
\]

\[
\ell'' = (p, \bot, \bot, P'', (S''_{\gamma})_{\gamma \in P''})
\]

\[
P'' = (P \setminus (P' \cap d')) \cup \{(d', c')\}
\]

\[
f < e \iff (d', c') \in P \land \exists (d'', c'') \in P \setminus P' (d'' \neq d' \land \mathcal{W}(d'') = \mathcal{W}(d'))
\]
Maintaining $K^2$

read event case 2

$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$

$\ell' = (p, d', c', P', (S'_\gamma)_{\gamma \in P'})$

$e < f$ implies $(d, c) \in P'$
Maintaining $K^2$

\[ \ell = (p, d, c, P, (S_\gamma)_{\gamma \in P}) \]
\[ \ell' = (p, d', c', P', (S'_\gamma)_{\gamma \in P'}) \]

\[ e < f \text{ implies } (d, c) \in P' \]
Maintaining $K^2$

Latest($\ell, \ell''$) = \{p\}

$\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$
$\ell' = (p, d', c', P', (S'_\gamma)_{\gamma \in P'})$

$\ell'' = (p, \bot, \bot, P'', (S''_\gamma)_{\gamma \in P''})$
$P'' = (P' \setminus (P' \cap d')) \cup \{(d', c')\}$

$e < f$ implies $(d, c) \in P'$

not case 1 and $(d, c) \in P'$ implies ...

not iff
Maintaining $K^2$

not (case 1 or case 2) implies $e \parallel f$

$$\text{Prim}(\downarrow g) = (\text{Prim}(\downarrow e) \cap \text{Prim}(\downarrow f)) \cup (\text{Prim}(\downarrow e) \setminus \downarrow f) \cup (\text{Prim}(\downarrow f) \setminus \downarrow e)$$

$$\text{Prim}(\downarrow e) \setminus \bigcup_{e' \in \text{Prim}(\downarrow e) \cap \text{Prim}(\downarrow f)} \text{Prim}(\downarrow e')$$
Maintaining $K^2$

Read event case 3

$h$ injective on $\text{Sec}(\downarrow e \cup \downarrow f)$

Not (case 1 or case 2) implies $e \parallel f$

$$\text{Prim}(\downarrow g) = (\text{Prim}(\downarrow e) \cap \text{Prim}(\downarrow f)) \cup (\text{Prim}(\downarrow e) \setminus \downarrow f) \cup (\text{Prim}(\downarrow f) \setminus \downarrow e)$$

$$P''' = (P \cap P') \cup \left( P \setminus \bigcup_{\gamma \in P \cap P'} S_\gamma \right) \cup \left( P' \setminus \bigcup_{\gamma \in P \cap P'} S'_\gamma \right)$$

$$\text{Prim}(\downarrow e) \setminus \bigcup_{e' \in \text{Prim}(\downarrow e) \cap \text{Prim}(\downarrow f)} \text{Prim}(\downarrow e')$$
Maintaining $K^2$

- **Read event case 3**
- $h$ injective on $\operatorname{Sec}(\downarrow e \cup \downarrow f)$
- $\operatorname{Latest}(\ell, \ell'')$ can be computed

Let:

- $\ell = (p, d, c, P, (S_\gamma)_{\gamma \in P})$
- $\ell' = (p, d', c', P', (S'_\gamma)_{\gamma \in P'})$
- $\ell'' = (p, \bot, \bot, P'', (S''\gamma)_{\gamma \in P''})$
- $P'' = (P''' \setminus (P' \cap d')) \cup \{(d', c')\}$

Not (case 1 or case 2) implies $e \parallel f$

- $P''' = (P \cap P') \cup \left( P \setminus \bigcup_{\gamma \in P \cap P'} S_\gamma \right) \cup \left( P' \setminus \bigcup_{\gamma \in P \cap P'} S'_\gamma \right)$. 

Latest event can be computed.
How to maintain the latest information using only finite set of messages?

- Is it even possible?
- At least in some cases?

- Synchronous communication [Zielonka87]
- Bounded channels [Mukund et al.03]

- Primary bounded